Tools to Support K–2 Students in Mathematical Argumentation

"I notice..."

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A yearlong professional development project investigated types of discourse and argumentation that students engage in, participation structures and routines that teachers can include to support students, and types of tasks that promote mathematical argumentation.

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Many mathematics lessons offer various opportunities for discussion and communication. In classrooms, students may give a numerical answer, tell the steps in a procedure, explain why they chose a strategy, ask a question of a peer or the teacher, conjecture about a pattern, or justify a claim. The different types of discussion and communication occurring in mathematics classrooms exist on a continuum; some types are more procedural, and some require higher levels of conceptual understanding. Although these various types of communicating have value, having opportunities to reach higher levels of discussion is important for students to further their conceptual understanding.

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One type of discourse that occurs in a classroom is mathematical argumentation, which is valuable in part because it promotes the development of conceptual understanding (Rumsey 2012). Students across K–grade 12 should have opportunities to "construct viable arguments and critique the reasoning of others" (CCSSI 2010, p. 6), and two ways to promote this in K–grade 12 classrooms are (1) scaffolding language and (2) strategically organizing ideas that are shared in the discussion. Ross, Fisher, and Frey found that—

> many young science students benefit from language frames to scaffold the use of academic language and vocabulary to formulate arguments and counterclaims (2009, p. 29)

In Japan, many teachers think carefully about how ideas from the discussions are organized and symbolically recorded (Stigler and Hiebert 1999). Both the scaffolding of language and strategic organizing of the shared mathematical ideas are tools that will be discussed in this article.

Research about upper elementary school students' argumentation exists (Rumsey and Langrall 2016; Rumsey 2012; Reid and Zack 2009; Stylianides 2007); less research is available about what argumentation looks like at the primary elementary school level (K–grade 2). Mathematical argumentation and meaningful discourse are important to include at all grade levels (NCTM 2014), but what does it look like in kindergarten through grade 2 classrooms? Early career K–grade 2 teachers in a yearlong project explored discourse and mathematical argumentation through monthly professional development (PD) sessions. The goal of the project was to develop an understanding of the types of K–grade 2 tasks that promote argumentation, the types of discourse and argumentation that K–grade 2 students engage in, and participation structures and routines that K–grade 2 teachers can include to support their students. On the basis of experiences from the project, we will highlight effective strategies for using language frames and public records to incorporate mathematical argumentation in the primary grades.

PD background and context

The teachers and students involved in this project were all part of the same school community on the West Coast of the United States. The school's population is about 600, including K–grade 8. All the teachers who were part of the PD were within their first three years of teaching and did not have a specific endorsement or degree in mathematics or mathematics education. The monthly PD sessions focused on mathematical argumentation and included discussions about current mathematics education literature, opportunities to explore and modify tasks to try in the classrooms, and video clubs (van Es and Sherin 2009).

Through the PD, the teacher participants contributed to a working definition of mathematical argumentation, identifying the essential elements as students engaged in the social and dynamic process of noticing mathematical ideas, sharing observations, generalizing the patterns, making conjectures or predictions, and justifying their thinking. The PD series topics included connections to ELA standards, representation-based proof, teacher questioning, modifying textbook tasks, and conjectures (see the **online more4U materials**). The series included the mathematical context of arithmetic properties, even and odd numbers, and comparing quantities. Although the topics and mathematical context varied, the structure of the PD sessions remained consistent. The components of each two-and-a-half-hour PD session were as follows:

- Video club, where teachers shared a short video recording of the previous argumentation task enacted in their classroom while the group analyzed the student thinking and the task
- Introduction of a new topic discussing and highlighting current research literature
- Defining new terminology and making connections to prior work
- Creating or modifying a new lesson to implement the new topic/discussion
- Reflective journals with prompts and questions

Leveraging ELA and mathematics standards

Because K–grade 2 teachers often teach all subject areas, finding ways to embed mathematical argumentation into the ELA standards (CCSSI 2010) is important, as is leveraging both sets of standards to make lessons more meaningful and of greater impact. While thinking about all the discourse opportunities that exist in a mathematics classroom, participants in the PD considered the ELA standards specifically for the category of "Speaking and Listening." For example, the ELA standards call for K–grade 2 students to "follow agreed-upon rules for discussions" (SL.K.1.A, SL.1.1.A, SL.2.1.A), build on others' comments (SL.1.1.B, SL.2.1.B), ask clarifying questions (SL.1.1.C, SL.2.1.C), and answer questions (SL.K.3, SL.1.3, SL.2.3).

As the teachers worked to support students' ability to "follow agreed-upon rules for discussions" and develop social norms, they encouraged students to use hand signals to show connections and agreement with peers, Students used a microphone during lesson share-outs to make sure that everyone could hear them, and teachers reminded listeners to turn their bodies toward the speaker. To encourage students to build on one another's comments, ask and answer questions, and engage with the speaker, the teachers asked students to retell their peer's steps in a short solution strategy; students asked clarifying questions of their peers, using agreed-on phrases. The focus on student language within the professional development led teachers to attend to student language throughout each lesson. For example, in a second-grade lesson, students interpreted one strategy for solving 21 + 17. The following excerpt is from that discussion.

Leo: So, what they're doing is they're adding the tens and the ones. So, they're putting them on top of each other, and then under the line is what the tens added and what the ones added. And then they equal to make the answer, so it equals thirty-eight.

Martin: They're decomposing the twenty-one into twenty and one, and the seventeen into ten and seven. So, they added twenty and ten to make thirty and [added] the one and seven to make eight. Thirty and eight is thirty-eight.

Teacher: I like how you took Leo's explanation and then added some more academic vocabulary in there by saying they *decomposed* the twenty-one and seventeen into twenty and one and [into] ten and seven.

In this example, the teacher recognized and made public the students' use of academic language in a way that provided explicit feedback of the specific desired language, making this visible to *all* students.

Participation structures and routines

As a result of the PD session, classroom teachers effectively incorporated language frames and public records into their mathematics instruction. These tools were instrumental in allowing students opportunities to meaningfully contribute to the discussion.

Language frames

Throughout the school year, each of the teachers regularly integrated language frames (Ross, Fisher, and Frey 2009) into their lessons to support students' language development. These frames were incorporated throughout the day and fit well with both the third of the Common Core (CCSSI 2010) Standards for Mathematical Practice (SMP 3) and the ELA standards. Language frames took the form of sentences posted on the wall with blanks for students to fill in during their mathematical conversations. For example, students may be asked to frame their conversation with such language frames as "I notice___" and "I wonder___." To introduce and encourage the use of language frames, one teacher asked, "I wonder what is the same about these two pictures and what is different? When I call you to come up, I want you to share, 'I notice ___.' I want you to share something that's different or something that's the same."

Teachers often used the language within the frames during lessons, within the questions they posed. Many students included that language in their responses. For example, one teacher asked, "Chris, what did you notice?"

Chris then responded, "I notice that there's two dogs and three cats."

Several instances occurred when teachers revoiced student responses, repeating what a student said and adding language from the frames. For example, Eliza stated, "They both have the same colors."

Eliza's teacher responded, "Oh, you noticed they both have the same colors."

The introduction of these frames allowed students to have a starting point to engage in the conversation and were also reminders for the teachers to use the same language. Many different types of language frames were introduced and were practiced regularly (see **fig. 1**).

As the school year progressed, the language frames were adapted to meet students' discourse needs. In grades 1 and 2, and midyear in kindergarten, the sentence frames were expanded to include predictions, justifications, and conjectures. For example, in some lessons, the language frame "I notice _____" transitioned to "I notice _____, and I think that this is because _____." As students became increasingly comfortable using the language frames, attention was paid to specificity of precise language. If a student shared, "I notice, *it* is _____," then the teacher prompted students to clarify what "it" was and to use specific language. In another example, the language frames and prompting by the kindergarten teacher helped a student to give a more detailed response:

Teacher: What's a way that these are the same? Beth?

Beth: One more

Teacher: Say, "I notice that ____."

Beth: I notice that Sean needs to have one more.

The use of these language frames in both student-to-teacher talk and student-to-student talk served as a foundation for conversations about student work. In another kindergarten lesson involving the commutative property of addition using dominoes, the language frames are evident in the classroom discussion:

Teacher: What do you notice about this one? What happened? Let's look again.

Heidi: I noticed that the other picture that we just saw had three here, and now this one has a three here and a one there. It's the same. One plus three still equals four even if you turn them around.

Teacher: Who can come up here and repeat what Heidi just said, because that was some pretty impressive math noticing. Bianca, do you know what Heidi just said? Come on up. Say, "I heard Heidi say____."

Bianca: I heard Heidi say that one and three still makes four even though they're flipped around.

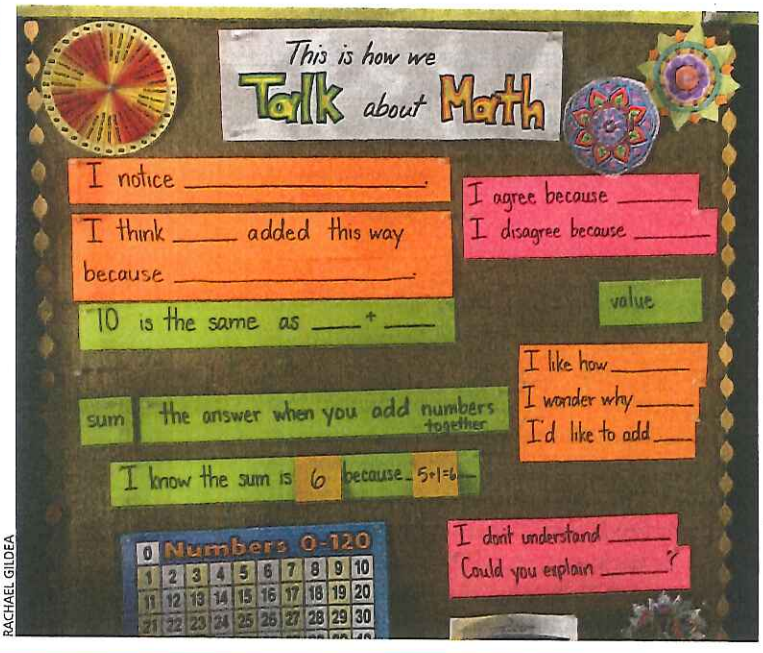
Teacher: Let's see if that's true. Oh, you guys! I love math so much.

Public records

Another beneficial participation structure that supported the K–grade 2 students in discourse and mathematical argumentation was making public records of the ideas they shared during whole-group discussions. Public records are written notes of ideas that every student can see and refer to during and after the lesson. In our classrooms, the public records took the form of chart paper, which could be saved and

FIGURE 1

At the end of the year, the first-grade reference board showed many different types of language frames, which were introduced early in the year and practiced on a regular basis.



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posted in the classroom. This was an advantage over using a whiteboard because the record became a more permanent part of the classroom. The public records of student thinking aided in increasing engagement and solidifying concepts, as it created a conceptual tool for students to refer to. Students could enter the conversation and see their peers' thinking. Public records also provided a way to show multiple solution strategies and build precise mathematical language. For example, in the first-grade classroom, students were often prompted at the beginning of the lesson to talk about what they noticed, using the language frame that had been established since the beginning of the school year. Students looked for patterns and shared what they noticed with the whole group, listening to their peers and showing connections using a silent hand signal. Students' ideas and solution strategies were recorded on the chart paper, using different colors with names attached to the ideas. Often, students used simple language, as opposed to precise academic language related to the content. The idea was to chart student thinking verbatim and build up to the academic language to help students see a bridge in language use. Students wanted to see their name on the chart, so they were motivated to actively explore what had already been written down and to listen to their peers. In the first-grade classroom, the public records from all the previous lessons were posted on a wall and became a reference for future lessons and a source of pride when students showed their parents around the classroom.

Some of the lesson structures used in the K–grade 2 classrooms that worked well for creating a public record included choral counts, number talks (Parrish 2010), open and true/false number sentences (Carpenter, Franke, and Levi 2003), and number sentence sets, such as these tasks:

Kindergarten task
What do you notice?
$2 + 2 = 4$
$2 + 3 = 5$
$3 + 3 = 6$
$3 + 4 = 7$
I notice _____

Grades 1–2 task
What do you notice?
$2 + 2 = 4$
$2 + 3 = 5$

$5 + 5 = 10$
$5 + 6 = 11$
I notice _____

Students were shown sets of number sentences and asked what they noticed and what conjectures they could make. Incorporated into the lesson was the language frame, "I notice ___." The task was set up and the numbers

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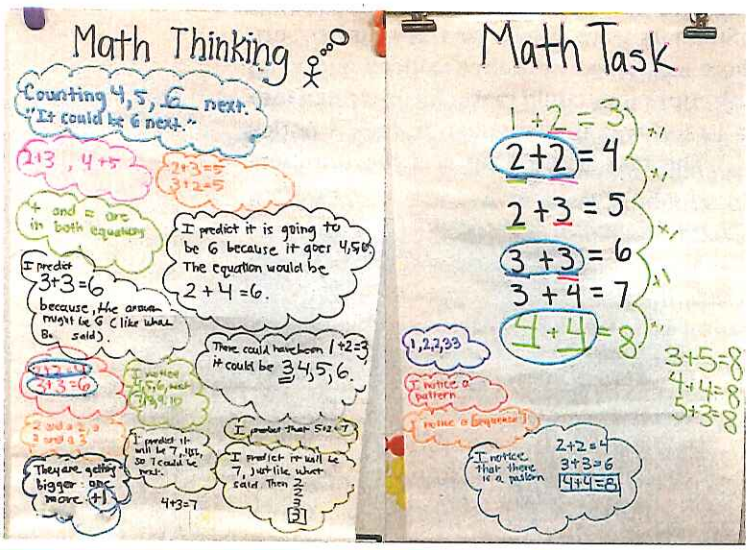
FIGURE 2

In each classroom, students noticed important mathematics and patterns, as in the kindergarten public record below. First- and second-grade students made predictions and conjectures about numbers in general on the basis of patterns they observed.

(a) Public records from all previous lessons, posted on a first-grade classroom wall, became a reference.

(b) Noticing patterns helped kindergartners make conjectures.



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were chosen so that students' observations explore addition, doubles, doubles plus one, and even and odd numbers while making predictions about what could come next in the set. In each classroom, students noticed important mathematics and patterns, and they made predictions (see **fig. 2**); in first and second grade, they were able to make conjectures about numbers in general on the basis of the patterns they observed (see **fig. 3**).

To prepare to incorporate public records into your lessons, sketching a general outline of what you would like the chart to look like at the end can be helpful before actually teaching the lesson. The purpose of this is to plan how to begin the lesson, ideas you anticipate that students will suggest, and questions you will ask to guide the lesson to the desired outcome or concept (Smith and Stein 2011).

K–grade 2 task characteristics

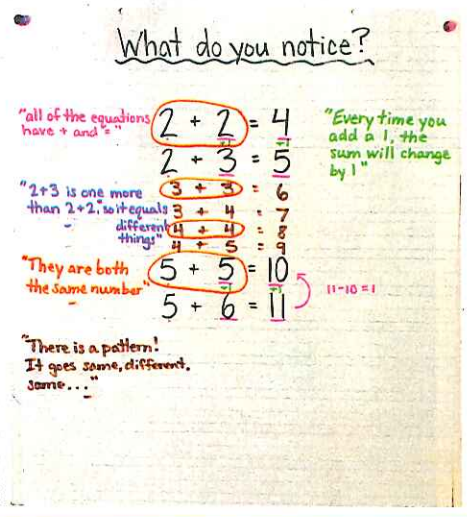
In addition to including participation structures and routines that support students and provide opportunities for them to meaningfully contribute to the discussion, certain types of tasks encourage mathematical argumentation

FIGURE 3

An important step included anticipating student noticings and wonderings during planning and having an idea of what the public record would look like.

(a) Student observations are recorded during whole-class discussions.

(b) Ideas shared publicly and recorded on charts help students make predictions and generalizations



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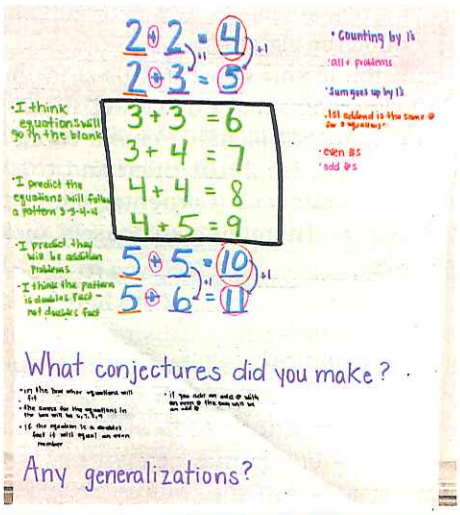

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and discourse at the K–grade 2 level. Open-ended tasks that included opportunities to notice, wonder, predict, and conjecture worked well because they allowed students to make and share many observations. When questions had more than one specific answer or strategy, students were tasked with solving the problem and determining a way to explain how they thought about the problem. This required students to understand different points of view when crafting their response and sharing with the group. For example, one task had many different possible observations and also had multiple deep connections to important mathematical concepts, such as addition and subtraction strategies, even and odd numbers, and number properties.

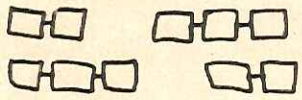
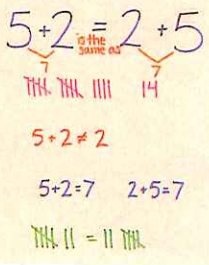
Teachers found that many opportunities for open-ended tasks encouraging discourse and mathematical argumentation occurred at the beginning of units. Even though the teachers were introducing new concepts, students felt comfortable sharing their opinions and responding to their peers' ideas while they built common, precise language that could be used throughout the unit. Including an open-ended task at the beginning of a unit can also serve as a formative assessment, and then future lessons can be built from observations and conjectures that students make.

As the teachers created and modified lessons, they were drawn to tasks that engaged with the important mathematical ideas. One task that was adapted across grade levels involved the commutative property of addition. During the first PD session, teachers used video available on the Illustrative Mathematics (IM) website to explore the idea of commutativity by thinking of how students would justify that 23 + 2 = 2 + 23 is true (Schifter, Bastable, and Russell 2008; IM 2016). The task was modified for each grade level (see **table 1**) and taught in September. Students had occasions to engage in meaningful mathematics standards, look for patterns, make observations, explore the

TABLE 1

In September, teachers explored the idea of commutativity and modified one task for each grade level.

K–grade 2 task adaptations for 23 + 2 = 2 + 23 with related Common Core State Standards

Kindergarten	First grade	Second grade
Using five linking cubes, the students individually determined different ways to decompose five into two groups. When students were back together as a whole group, the teacher made connections between the solutions related by the commutative property. For example, 2 + 3 and 3 + 2. 	Show your thinking: 5 + 2 = 2 + 5 Is this true or false? Explain your thinking. Public record of the whole-group discussion: 	True or False? 23 + 2 = 2 + 23 Explain how you know. As a follow-up activity, students were then given the following task: True or false? 146 + 72 = 72 + 146 Explain how you know.

Related math content standards

K.OA.A.3	1.OA.B.3	2.NBT.B.7
K.OA.A.5	1.OA.D.7	

commutative property of addition, explain their thinking, communicate mathematical ideas as a whole class, and justify their reasoning.

Discussion

Students in K–grade 2 are capable of deep mathematical thinking and communicating. Although research exists on mathematical argumentation at the upper elementary school grade levels (Rumsey and Langrall 2016; Rumsey 2012; Reid and Zack 2009; Stylianides 2007), this professional development project sought to expand our understanding of the types of discourse and argumentation that K–grade 2 students engage in during math class, participation structures and routines that K–grade 2 teachers can include to support students, and K–grade 2 tasks and topics that promote argumentation. By leveraging ELA and mathematics standards (CCSSI 2010), participation structures and routines that supported students in both language arts and mathematics could be incorporated. Because students participated in classrooms where discourse and mathematical argumentation were encouraged throughout the entire school year, by the end of the academic year, they engaged in advanced conversations that included predictions, conjectures, and justifications. We hope that sharing the results and reflections from our yearlong project will encourage and support other educators to include discourse and argumentation at the K–grade 2 level.

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